Frequency Domain Measures:
The Fourier Transform, the Lomb Periodogram, and Other Methods

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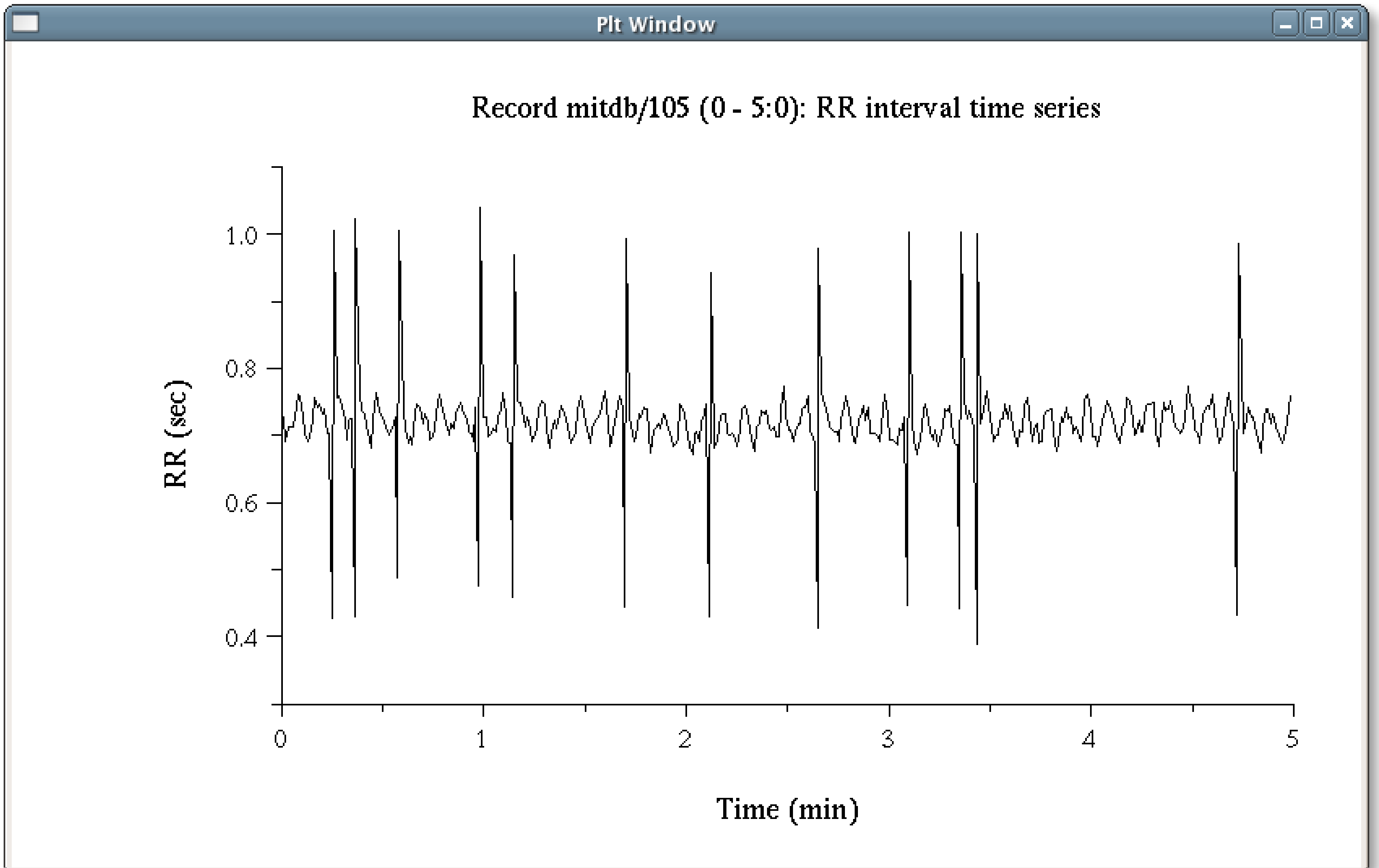
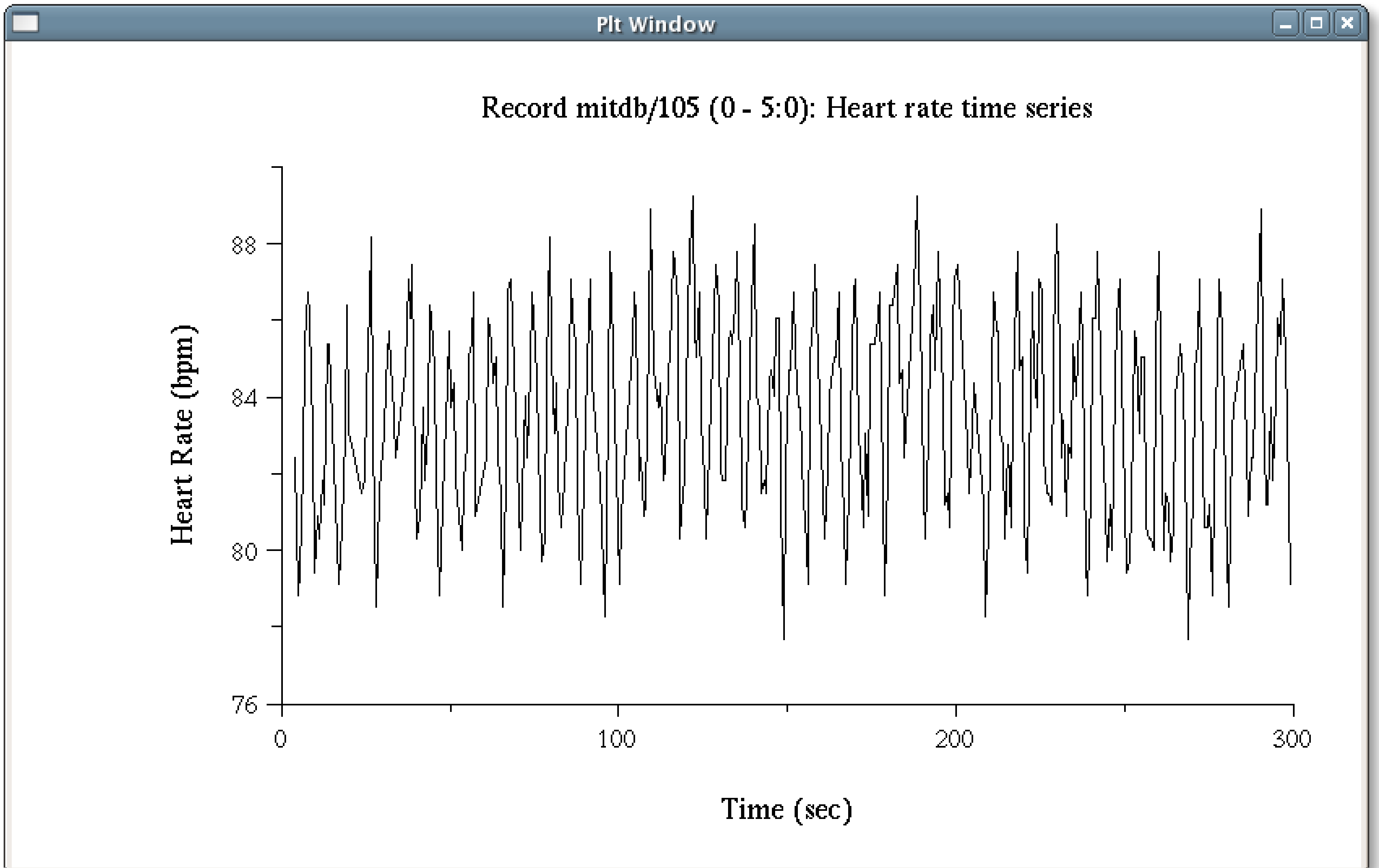
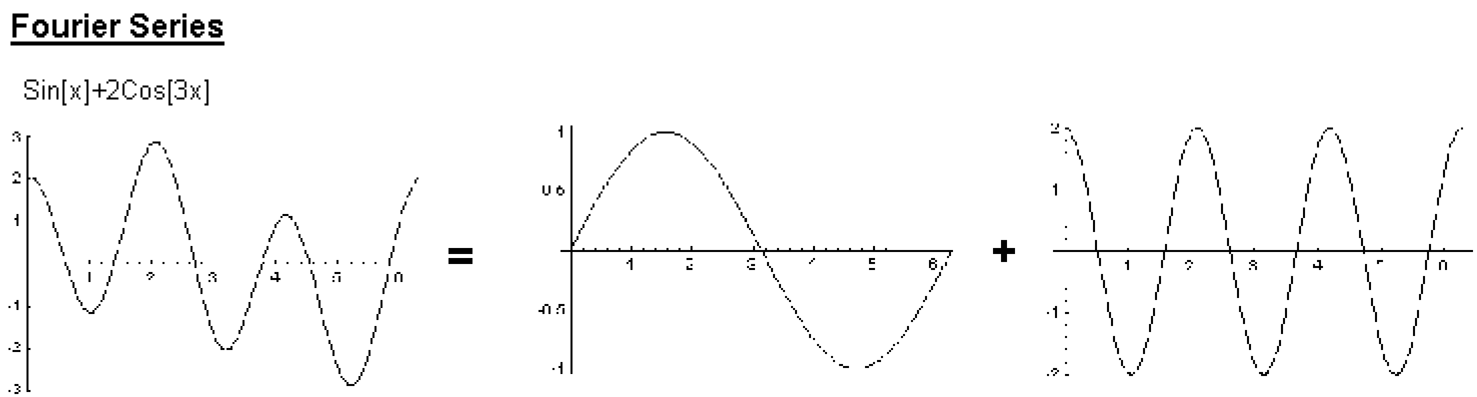
"Fourier's theorem tells us that every curve, no matter what its nature may be, or in what way it was originally obtained, can be exactly reproduced by superposing a sufficient number of simple harmonic curves - in brief, every curve can be built up by piling up waves."

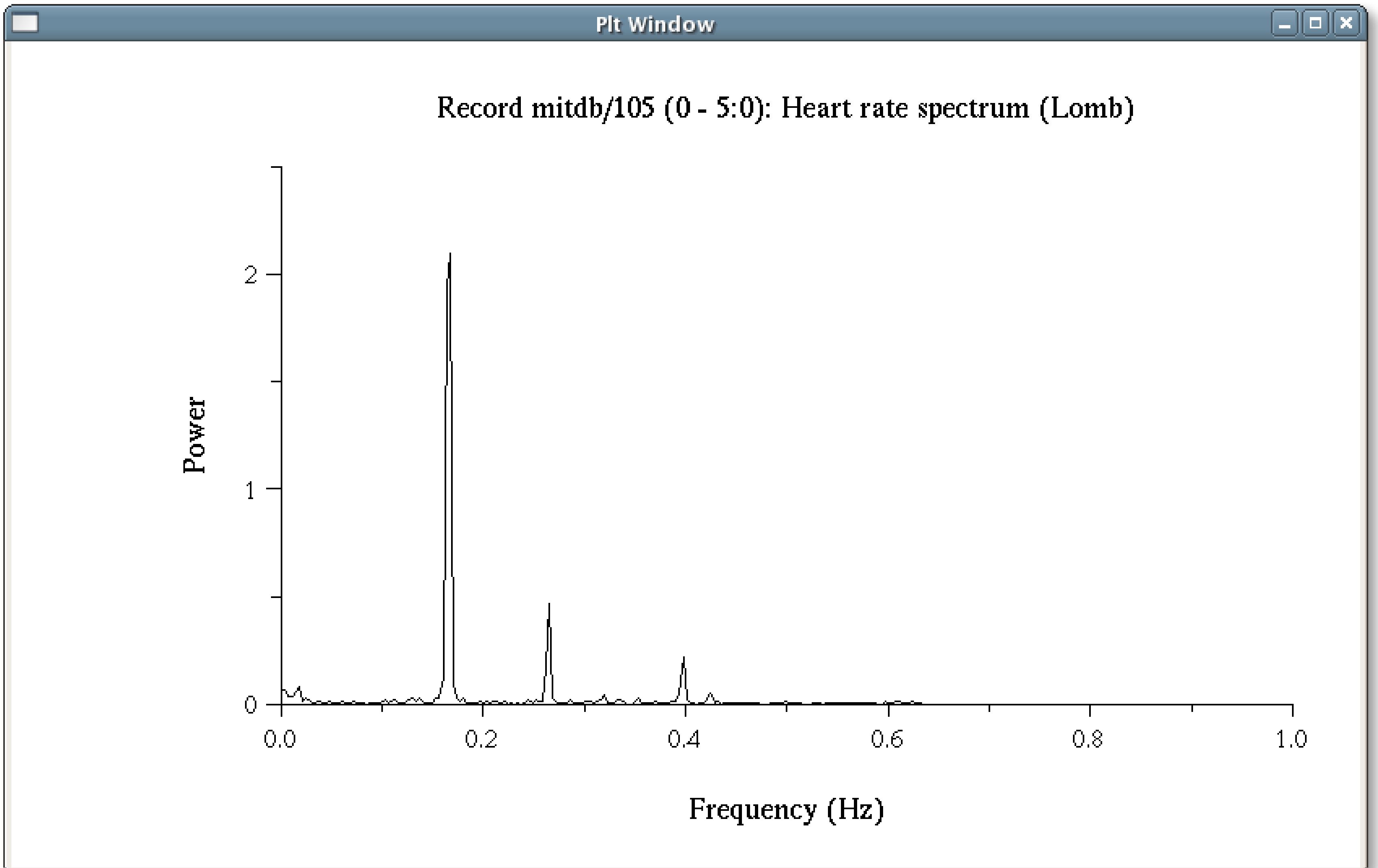
Sir James Jeans (1877-1946)


Plt Window
Record mitdb/105 (0 - 5:0): RR interval time series
RR (sec)
1.0
0.8
0.6
0.4
0
1
2
3
4
5
Time (min)

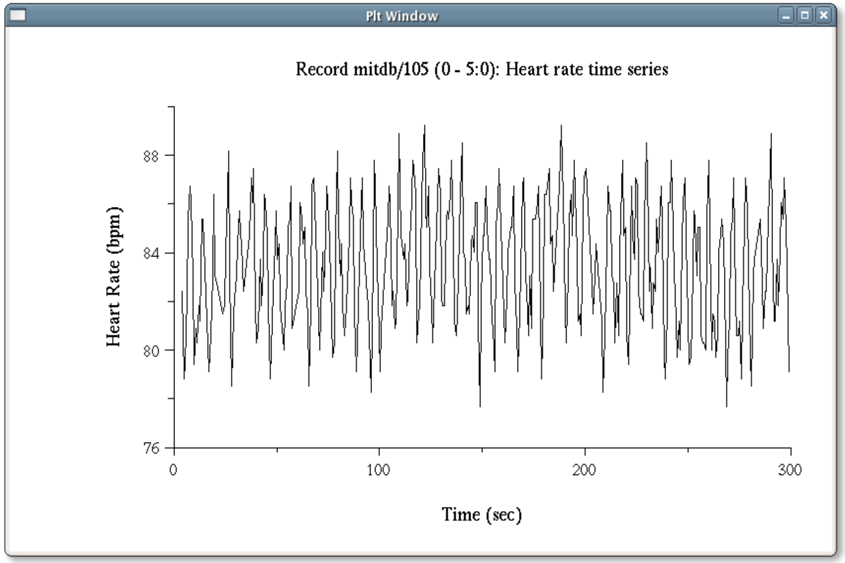

Plt Window
Record mitdb/105 (0 - 5:0): Heart rate time series
Heart Rate (bpm)
88
84
80
76
0
100
200
300
Time (sec)

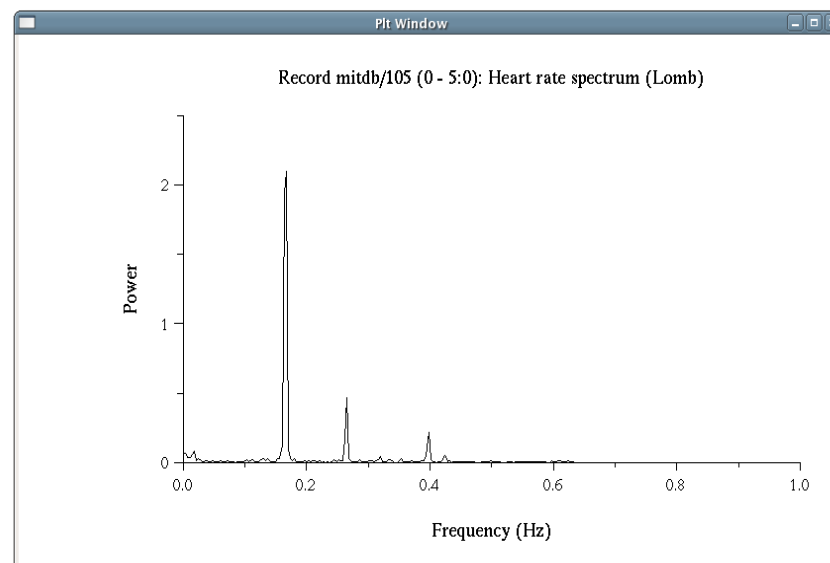


http://www.kopernekus.com/images/fourier_img1.gif


Plt Window
Record mitdb/105 (0 - 5:0): Heart rate spectrum (Lomb)
Power
2
1
0
0.0
0.2
0.4
0.6
0.8
1.0
Frequency (Hz)

Time and frequency domains are equivalent



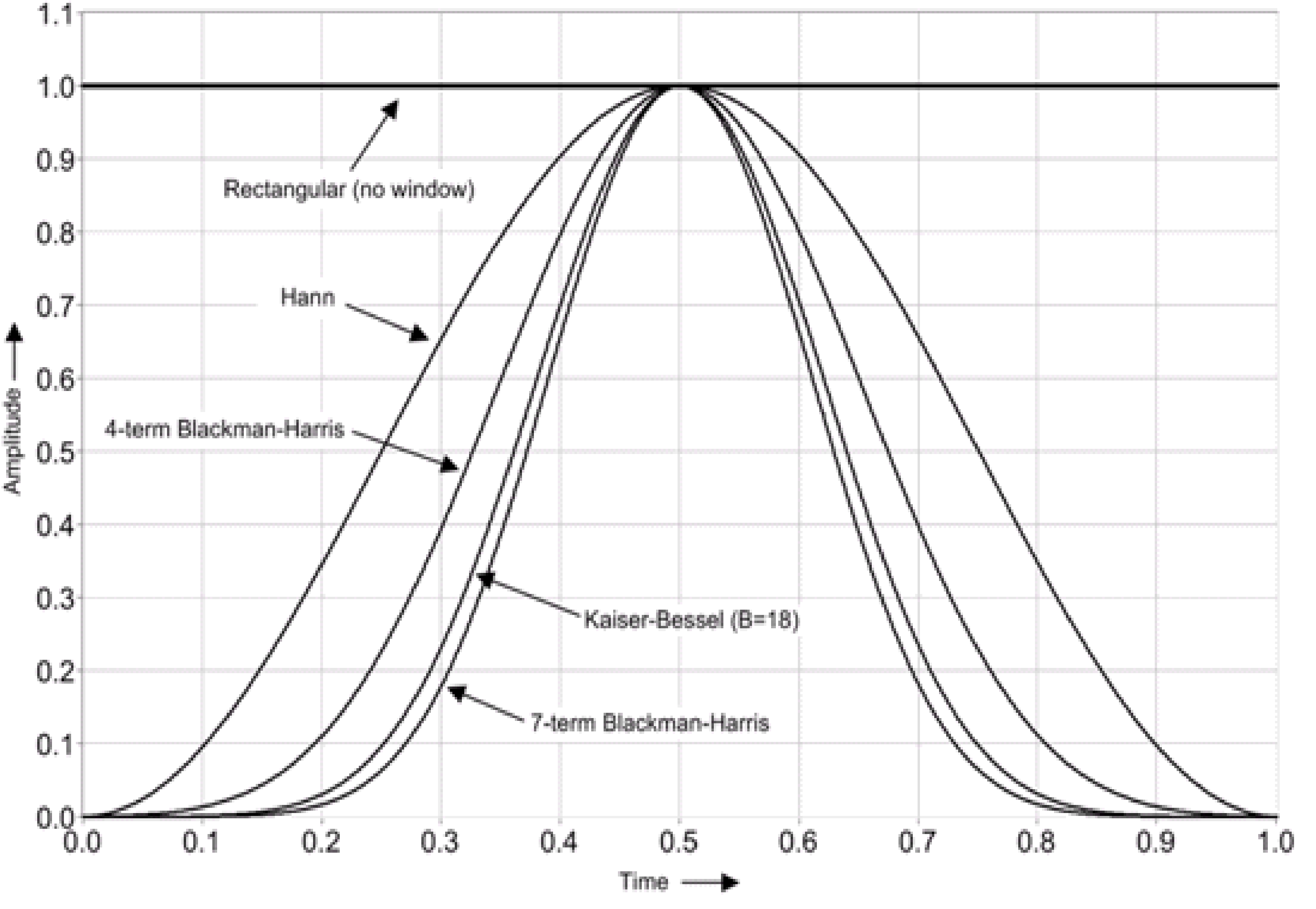


Information is neither created nor destroyed by changing domains

Harmonic analysis of nonperiodic functions

- Only periodic functions have Fourier series representations
- HR time series are not periodic!
- Solution: stitch together multiple copies of nonperiodic series end-to-end
- Use window functions to avoid discontinuities at the “seams”

Examples of window functions



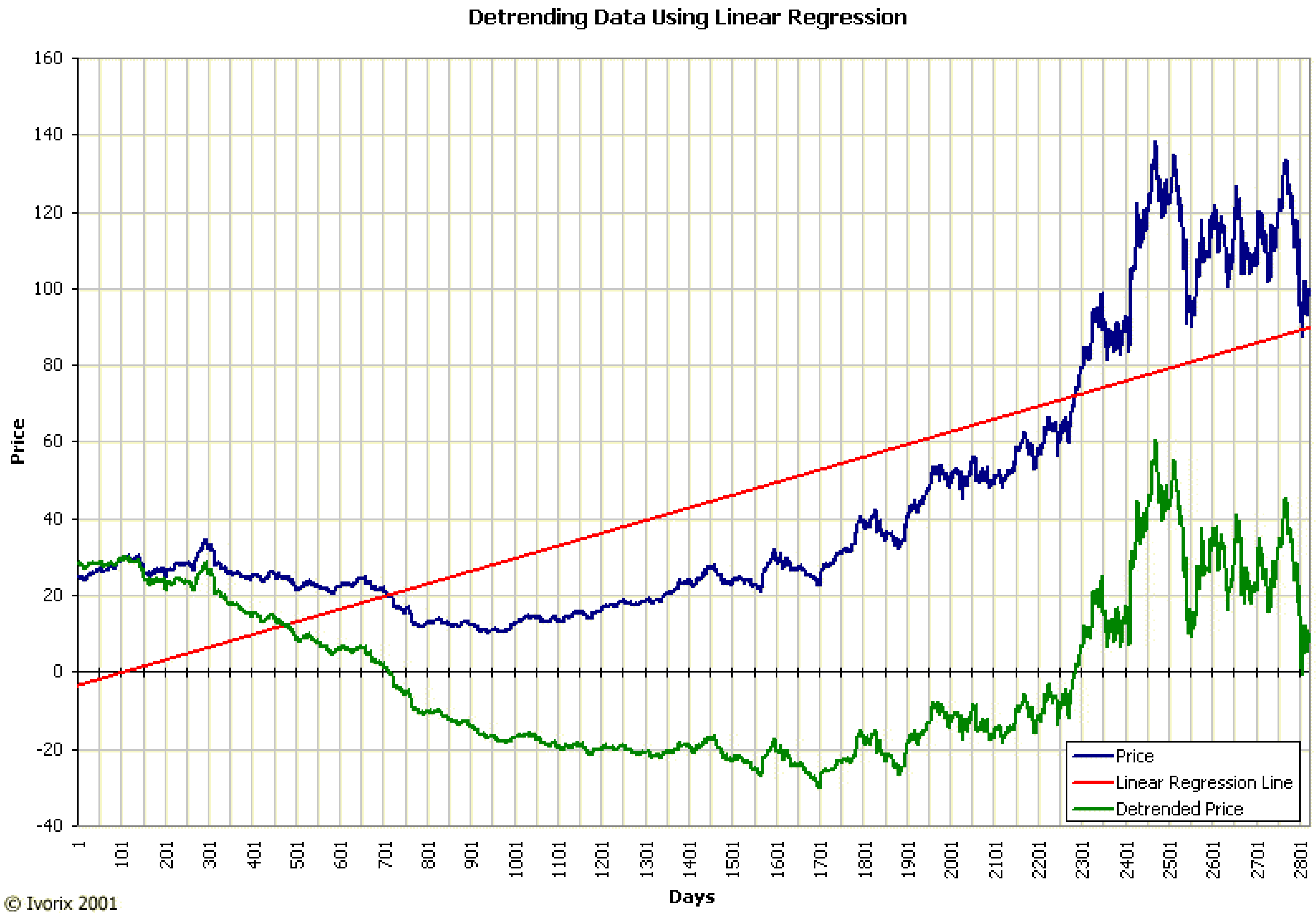
http://zone.ni.com/devzone/conceptd.nsf/webmain/E0F0B1FFE86A14A986256F2E0005AF4D

Using window functions

- Subtract the mean from the input series ("zero-mean" the time series)
- Stretch the window to match the input
- Multiply the input by the window

Stationarity and detrending

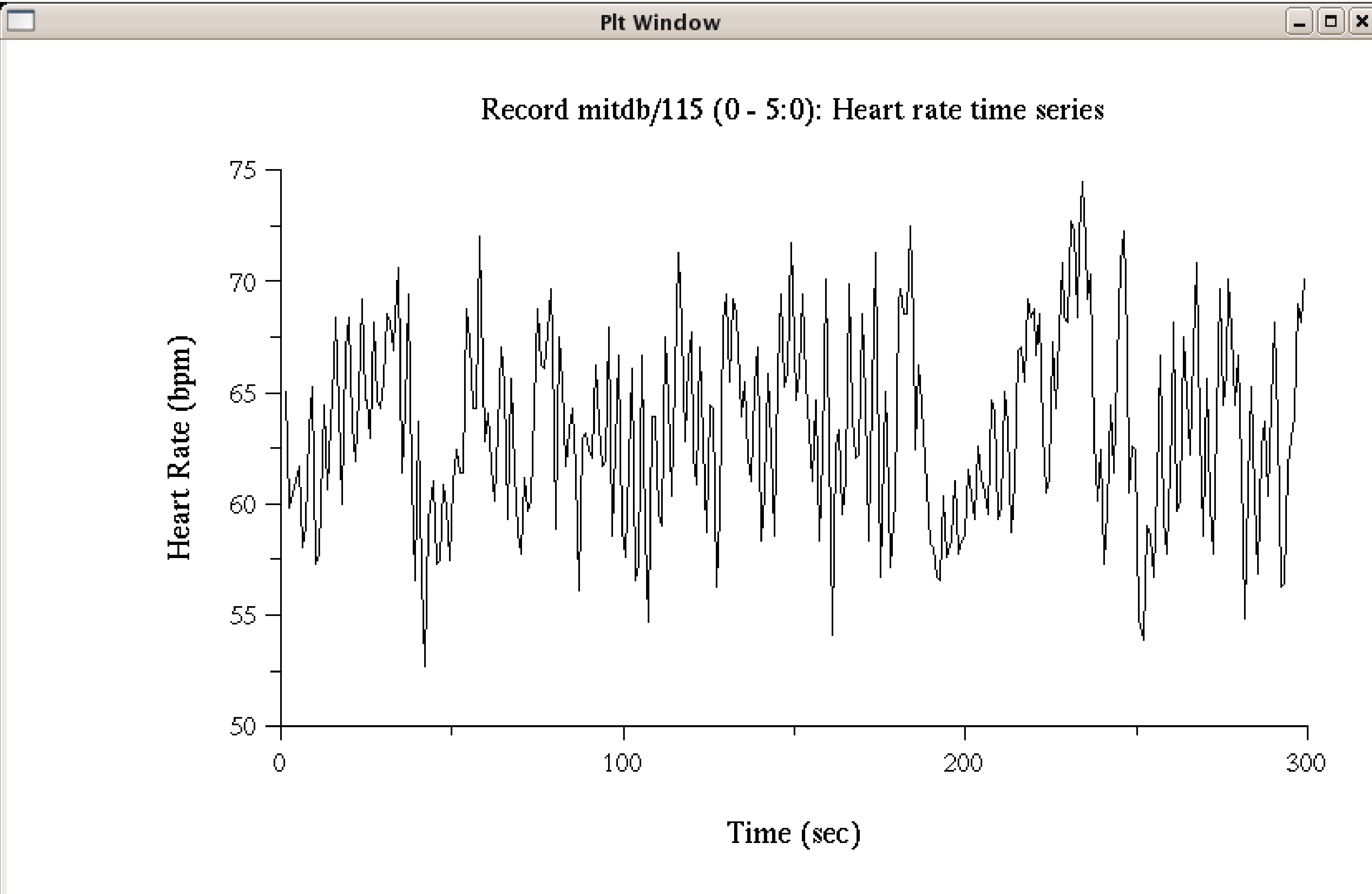
- Input must be stationary (no significant changes in means or sample variances)
- In some cases, detrending (subtraction of a fitted line) may be acceptable



http://www.cbi.dongnocchi.it/glossary/Gif/Aliasing.gif

Uniform and nonuniform sampling

- In HRV analysis, nonuniform sampling is a given!
- All Fourier methods and AR methods require uniform sampling
- Option 1: Pretend sampling intervals are fixed
  => “beat-number domain”, not frequency
  domain analysis
- Option 2: Resample at uniform intervals



Plt Window
Record mitdb/115 (0 - 5:0): Heart rate time series
Heart Rate (bpm)
75
70
65
60
55
50
0
100
200
300
Time (sec)

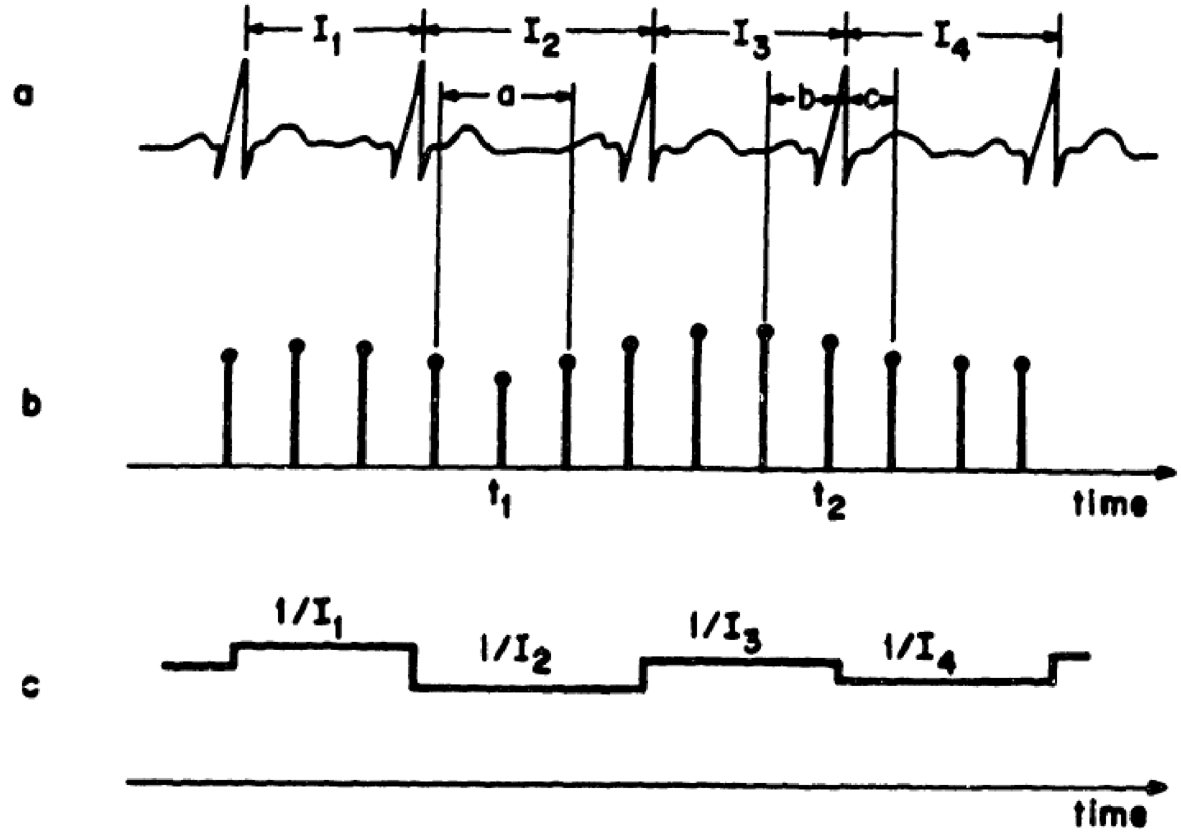


Figure 4.2. a. A segment of an ECG signal. b. The heart rate samples corresponding to the ECG signal in (a), determined using our algorithm. The number of RR intervals within the local window centered at t_1 is a/I_2, and at t_2 is $b/I_3 + c/I_4$. The value of the heart rate at each sample point is taken to be the number of intervals that fell within the local window centered at that point divided by the width of the window, as described in the text. c. The corresponding instantaneous heart rate signal. The value held during each interval is the reciprocal of the duration of that interval.

Berger, RD. 1987. Analysis of the cardiovascular control system using broad-band stimulation

Handling missing and ectopic beats

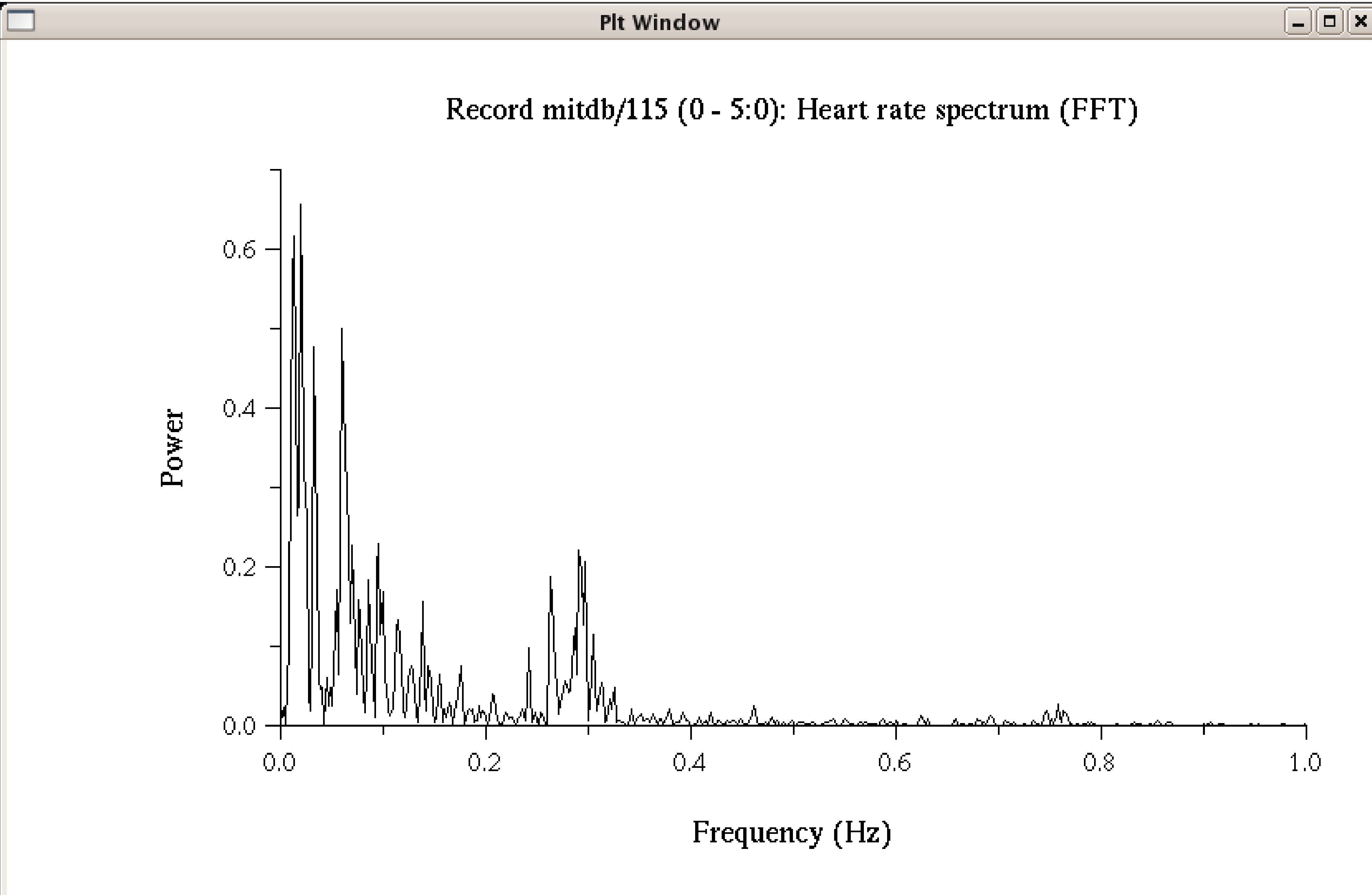
- Use “perfect” data only
- Delete interruptions and concatenate surrounding data
- Interpolate missing sinus beats

Discrete Fourier Transform (DFT)

- Produces discrete spectrum (magnitudes defined only for discrete frequencies)
- Highest frequency (the Nyquist frequency) is half of the sampling frequency
- Original DFT algorithms are slow

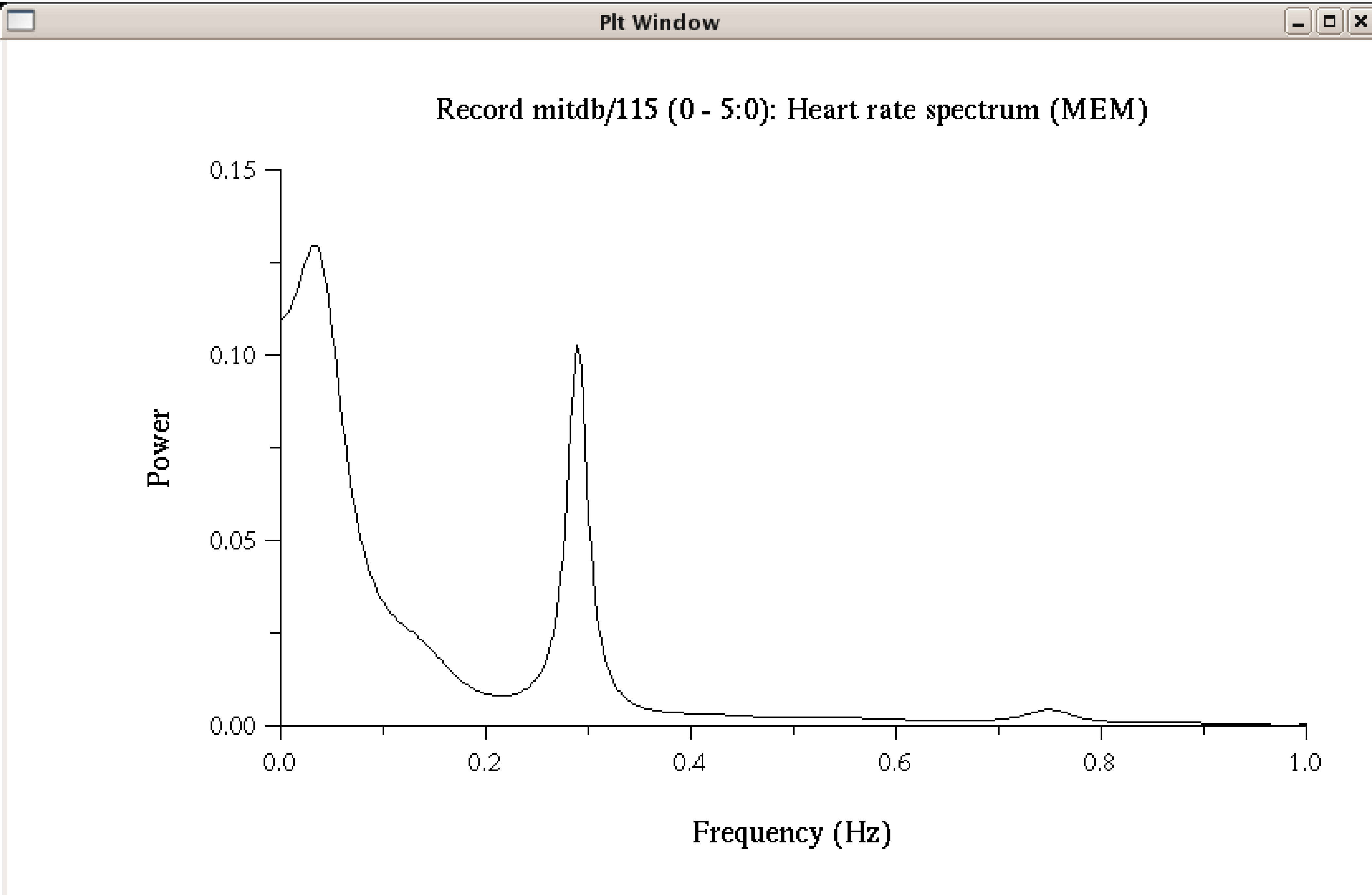
Fast Fourier Transform (FFT)

- Cooley and Tukey (1965)
- Not approximate – faster and better
- Most efficient when input series length is exactly a power of two
- Zero-pad series of other lengths (after zero-meaning) to the next largest power of two


Plt Window
Record mitdb/115 (0 - 5:0): Heart rate spectrum (FFT)
Power
0.0
0.2
0.4
0.6
0.0
0.2
0.4
0.6
0.8
1.0
Frequency (Hz)

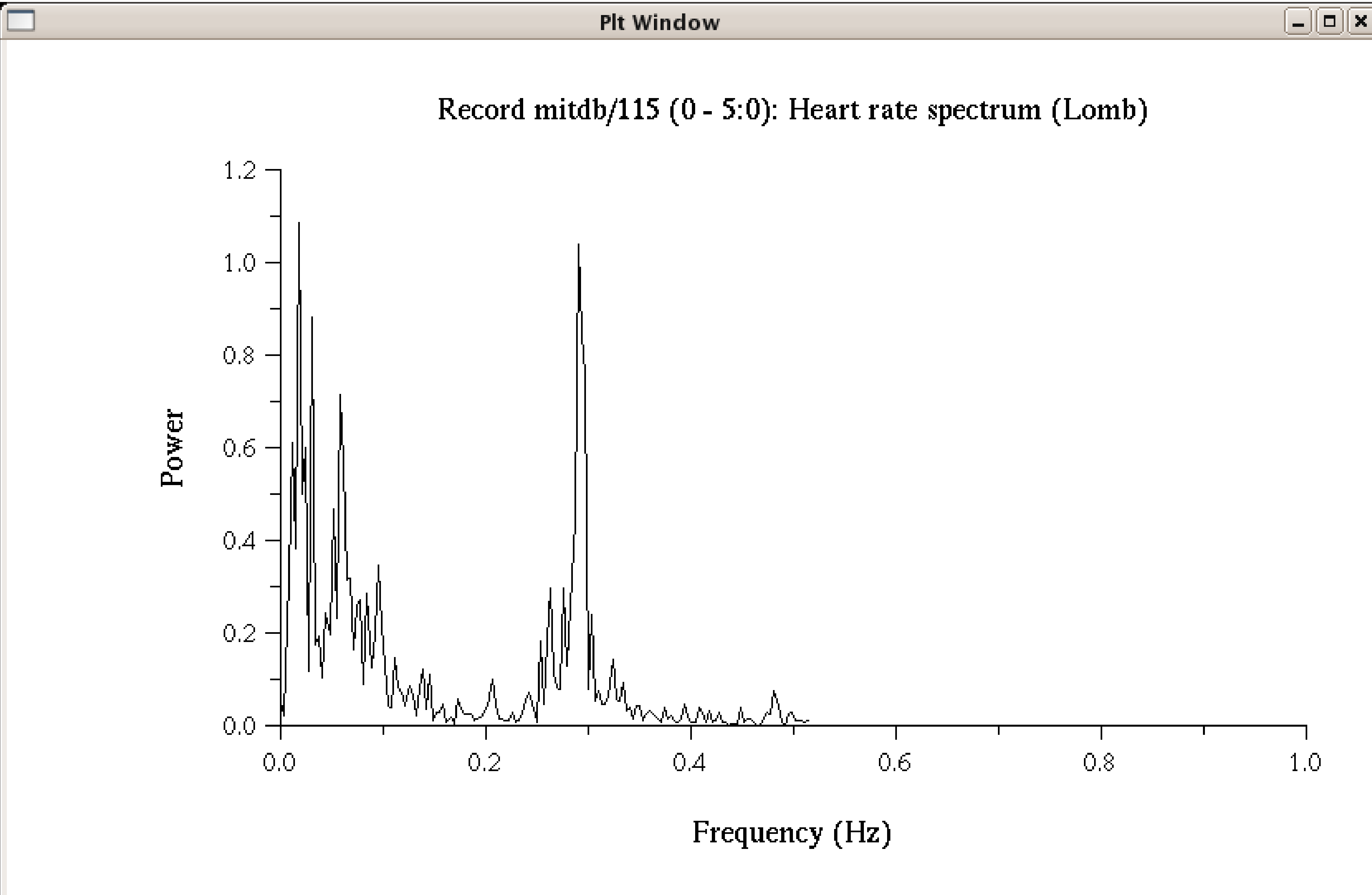
Autoregressive (AR) spectral density estimation

- Also called maximum entropy (MEM) or “all-poles” method
- Fits “line” features well
- AR spectra look clean and uncluttered
- AR spectra reflect not only the input, but also assumptions about its complexity
- Use with care!


Plt Window
Record mitdb/115 (0 - 5:0): Heart rate spectrum (MEM)
0.15
0.10
0.05
0.00
Power
0.0
0.2
0.4
0.6
0.8
1.0
Frequency (Hz)

The Lomb periodogram

- Lomb, 1976; Press & Rybicki 1989
- Fit a sine to the available data
  => one point in the spectrum
- Subtract the fitted sine wave
- Repeat to get another point in the spectrum, etc.


Plt Window
Record mitdb/115 (0 - 5:0): Heart rate spectrum (Lomb)
Power
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
Frequency (Hz)

Common HRV Frequency Domain Measures

Total HRV power:	0 - 0.5 Hz
ultra-low frequency (ULF) power:	0 - 0.0033 Hz
very low frequency (VLF) power:	0.0033 - 0.04 Hz
low frequency (LF) power:	0.04 - 0.15 Hz
high frequency (HF) power:	0.15 - 0.4 Hz
very high frequency (VHF) power:	0.4 - 0.5 Hz

LF/HF ratio

Units of HRV power

- HRV power spectra: If the HR time series units are
  beats / second
  the power spectrum units are
  $seconds^2$ / Hz
  and power in a band is in units of
  $seconds^2$

- Power in a band is often converted to units of
  ms^2

- Normalized units are dimensionless; power in a band can be expressed as a fraction of total power

"Mathematics compares the most diverse phenomena and discovers the secret analogies that unite them."

Jean Baptiste Joseph Fourier (1768-1830)